TAKE-HOME MIDTERM ---- SUGGESTED ANSWERS

All examinations are open-book, open-notes. There is no time limit other than the due date. Confidentiality is required during examinations. Please strictly observe academic integrity. Examinations should be your own personal work. During examinations, other people (classmates, friends, professors --- except Troy and Prof. Starr) are CLOSED; do not discuss examination materials until after the exam has been collected. If you have questions, e-mail them to Prof. Starr at rstarr@ucsd.edu.

Answer any three (3) questions. They count equally. If you write more than three answers, designate which three you want to be counted.

1. Consider a three person pure exchange economy. There are two commodities x and y. Household 1 has endowment $r^1=(r^1_x, r^1_y) = (10, 2)$, household 2 has endowment $r^2=(r^2_x, r^2_y)=(6, 14)$, household 3 has endowment $r^3=(r^3_x, r^3_y)=(8, 8)$. All households have the same utility function on X^i= the nonnegative quadrant of R^2, $u^i(x, y) = \sup [x, y]$, where sup stands for supremum or maximum.

(i) Demonstrate that this economy has no competitive equilibrium.

(ii) Is this a counterexample to the existence of general equilibrium theorem 7.1? If so, explain why. If not, explain how this example fails to fulfill the assumptions of that theorem in a way that causes non-existence of equilibrium.

***Suggested Answer*: (i)** At any price vector (½ + ε , ½ - ε) , ½ >ε >0, there is excess demand for y. At any price vector (½ - ε , ½ + ε) there is excess demand for x. At (½ , ½) , 1's demand is either (12, 0) or (0, 12), 2's is either (20, 0) or (0, 20), 3's is either (16, 0) or (0, 16). There is no combination of these demands, one for each household, so that demands total (24, 24) which is total supply. Hence there is no market clearing price.

(ii) The preferences displayed are nonconvex, failing C.VI and C.VII. Demands jump from one side of the consumption set to the other, an apparent discontinuity, so the assumptions of the theorem are not fulfilled. This is not a counterexample.

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2. Consider a two-person, two-commodity pure exchange economy (an Edgeworth box). Household 1 has endowment $r^1=(r^1_x, r^1_y) = (5, 0)$; household 1 owns only x. Household 2 has endowment $r^2=(r^2_x, r^2_y)=(5, 10)$. Household 1 has preferences summarized by the utility function, $u^1(x,y) = x + y$. Household 2 has preferences summarized by the utility function $u^2(x, y) = y$. Household 2 does not value x. Preferences in this economy are convex (fulfilling C.VI but not C.VII) but not strictly convex, but that is <u>not</u> the problem. Consider $p^* = (\varepsilon , 1-\varepsilon)$ for $1>\varepsilon>0$. p* cannot be an equilibrium, since it generates an excess supply of x. But at $p^o=(0, 1)$ there is no equilibrium either, since there is an excess demand for x. How can this observation be consistent with the existence of general competitive equilibrium theorem, theorem 7.1? Is one of the assumptions (aside from C.VII) of 7.1 not fulfilled? Explain.

Suggested Answer: Household 1 does not fulfill C.VIII (adequacy, positivity, of income). At price vector p^o , 1's income is zero, placing it's income at the minimum level in it's possible consumption set (R^2_+). This opens the possibility of discontinuous demand, which has occurred in this example.

3. Consider an Edgeworth box economy. Household 1 has endowment $r^1=(r^1_x, r^1_y) = (5, 5)$, household 2 has endowment $r^2=(r^2_x, r^2_y)=(10, 10)$. Household 1 has preferences summarized by the utility function, $u^1(x,y) = xy$. Household 2 has preferences summarized by the utility function $u^2(x, y) = \inf[xy, 64]$ where inf stand for infimum or minimum. That is, household 2 is satiated with consumption when his utility level gets to 64.

- **(i)** Demonstrate that this economy has a competitive equilibrium at prices (½ , ½).
- **(ii)** Demonstrate that the equilibrium allocation in part (i) is Pareto inefficient.
- **(iii)** Is this a counterexample to the First Fundamental Theorem of Welfare Economics, Theorem 12.1? If so, explain why. If not,

explain how this example fails to fulfill the assumptions of that theorem in a way that permits an inefficient equilibrium allocation.

Suggested Answer: **(i)** 1's demand is (5, 5), 2's is (10, 10). The market clears. **(ii)** 1: (7, 7), 2: (8, 8) is Pareto preferable. **(iii) 1FTWE** depends on monotonicity, C.IV. But 2 is satiated at (8,8), hence the inefficiency is possible in equilibrium without violating 1FTWE.

4. Explain the significance of the theorems demonstrating existence of general competitive equilibrium, Theorems 1.2 and 7.1. Why should economists be interested in general equilibrium? Why should they be interested in sufficient conditions for its existence?

Suggested Answer: Market clearing equilibrium is a standard solution concept in economics --- often applied to individual markets in isolation (partial equilibrium). But since there are strong interactions among markets it is important to know whether all markets can clear simultaneously fully taking into account the interactions among them. That's general equilibrium. Understanding the sufficient conditions for existence of a general equilibrium means knowing when we can reliably expect market clearing to occur.